DEUTSCHER KALIBRIERDIENST DKD

Guideline DKD-R 6-2

Calibration of Measuring Devices for Vacuum

Part 2

Measurement Uncertainties

Published by the Accreditation Body of the Deutscher Kalibrierdienst (DKD) at the Physikalisch-Technische Bundesanstalt in co-operation with its Technical Committee "Pressure and Vacuum".

Deutscher Kalibrierdienst (DKD)

The DKD comprises calibration laboratories in industrial enterprises, research institutes, technical authorities, inspection and testing institutes. They are accredited and supervised by the DKD. They calibrate measuring instruments and material measures for measurands and measurement ranges specified within the scope of accreditation. The DKD calibration certificates issued by these laboratories prove traceability to national standards as required in the ISO 9000 family and ISO/IEC 17025.

Calibrations carried out by DKD laboratories ensure that the user may rely on measurement results. They increase the customers' confidence and competitiveness on the national and international markets and serve as a metrological basis for the inspection of measuring and test equipment within the framework of quality assurance measures.

Calibrations offered by the DKD cover electrical measurands, length, angles and other geometrical quantities, roughness, coordinate and form measuring techniques, time and frequency, force, torque, acceleration, pressure, flowrate, temperature, humidity, medical measurands, acoustic measurands, optical measurands, ionizing radiation and other measurands.

Publications: see Internet

Address:

Deutscher Kalibrierdienst at the
Physikalisch-Technische Bundesanstalt

Bundesallee 100	D-38116 Braunschweig
P.O. Box 33 45	D-38023 Braunschweig
Office telephone	+49 531 592 1901
Fax	+49 531 592 1905
E-Mail	dkd@ptb.de
Internet	www.dkd.info

Foreword

DKD Guidelines are application documents for the general criteria and procedures which are laid down in DIN EN ISO/IEC 17025 and DKD publications. The DKD Guidelines describe technical and organizational processes serving the calibration laboratories as a model for laying down internal procedures and regulations. DKD Guidelines can become an integral part of quality manuals of calibration laboratories. The application of the Guidelines supports equal treatment of the devices to be calibrated at the different calibration laboratories and improves the continuity and verifiability of the work of the calibration laboratories.

The DKD Guidelines will not impede the further development of calibration procedures and sequences. Deviations from guidelines and new methods are permitted in agreement with the Accreditation Body if they are justified by technical aspects.

The present Guideline was prepared by the Technical Committee "Pressure and Vacuum" in cooperation with the PTB and adopted by the Advisory Board of the DKD. With its publication it is binding for all DKD calibration laboratories unless separate procedural instructions approved by the Accreditation Body are available.

This document is a translation of the German Guideline R 6-2. In case of any disputes the respective German version is binding.

Contents

1 Scope of application

This part of the Guideline deals with the determination, quantification and budgeting of measurement uncertainties for the calibration of vacuum gauges in accordance with this Guideline.

2 Definition

The measurement uncertainty[1] is defined as the parameter which is stated together with the measurement result, i.e. which is attributed by the measurement to the measurement result, and characterizes the range of values which can reasonably be assigned by the measurement to the measurand.

3 Model

The uncertainty analysis is on principle carried out in accordance with the specifications given in publication DKD-3.

For the calibration of vacuum gauges the sum/difference model has proved to be suitable provided the influence quantities are uncorrelated.

3.1 Sum / difference model

$$Y = X + \sum_{i=1}^{N} \delta X_i$$

Y result quantity
X input quantity/quantities determining the value
δX_i unknown deviation(s)

In the calibration of vacuum gauges, the quantity to be determined is in most cases the deviation Δp. The general sum/difference model thus becomes

$$\Delta p = p_{Anzeige} - p_{Normal} + \sum_i \delta pi$$

with

p_{Anzeige} indication of vacuum gauge
p_{Normal} value of standard
δp_i further unknown deviations

In vacuum measuring technology, it is preferred for reasons of clarity to subdivide the unknown deviations according to calibration item, standard and method.

$$\Delta p = p_{KG} - p_N + \delta p_V$$

with

p_N conventional true value of standard (section 4.1)
p_{KG} (measured) value of calibration item (section 4.2)
δp_V differences due to calibration method (section 4.3)

[1] For the terminology, see DIN EN 13005

"Conventional true value of standard" means that all known corrections have been applied to the indicated value (deviation of indication according to calibration certificate, offset, temperature correction, etc.).

"(Measured) value of calibration item" means that the offset and, for example, the height correction have been applied to the indicated value.

4 Calculation of measurement uncertainty

For the calculation of the expanded uncertainty U the following equation is valid:

$$U = k \cdot \sqrt{u_N{}^2 + u_{KG}{}^2 + u_V{}^2}$$

with

k coverage factor
u_N uncertainty contribution of corrected pressure indication of standard
u_{KG} uncertainty contribution of corrected pressure indication of calibration item
u_V uncertainty contribution of calibration method (procedure)

To simplify things, the indices of the pressure from the model equation have been used here (in the place of u by the measurement u_{PN} : u_N).

The sensitivity coefficients generally have the magnitude one, i.e. the uncertainty contributions to be attributed to the result are of the same magnitude as the standard uncertainties of the input quantities.

4.1 Uncertainty contribution u_N of standard

The standard uncertainties of the measurement values of the standard are given by the calibration of the standard. When it is used at the calibration laboratory, effects depending on the long-time instability and the respective conditions of calibration are, however, to be taken into account.

For the (measured) value of the standard, the following model is typically valid:

$$p_N = p_{Anz,N} - p_{Offs,N} + \delta p_{D,N} + \delta p_{Cal,N} + \delta p_{L,N} + \delta p_{T,N} + \delta p_{S,N}$$

with

p_N conventional true value
$p_{Anz,N}$ indication
$p_{Offs,N}$ offset (zero deviation) of the standard. Note: If the offset is deducted or adjusted to zero in the device itself, $p_{offs,N} = 0$.
$\delta p_{D,N}$ deviation of offset due to drift
$\delta p_{Cal,N}$ correction according to calibration certificate (deviation)
$\delta p_{L,N}$ deviation due to long-time instability (assumption: in most cases, $\delta p_L = 0$)
$\delta p_{T,N}$ deviation of indication due to temperature influence at the calibration laboratory
$\delta p_{S,N}$ deviation due to other influences (e.g. inclination of device, etc.)

For the standard uncertainty to be attributed to the values of the standard, the following relation is valid:

$$u_N = \sqrt{u_{Anz,N}{}^2 + u_{Offs,N}{}^2 + u_{D,N}{}^2 + u_{Cal,N}{}^2 + u_{L,N}{}^2 + u_{T,N}{}^2 + u_{S,N}{}^2}$$

with

$u_{Anz,N}$ uncertainty component of the values of the standard due to insufficient repeatability at the time of calibration of the calibration item (dispersion of measurement values about a mean value, including dispersion due to digitizing, resolution, etc.)
Note: There often is only one measurement value. In this case, the uncertainty component must be estimated from empirical values (including uncertainty component due to digitizing, offset variations, etc.).

$u_{Offs,N}$ uncertainty of the offset values at the time of measurement of the offset (without repeatability of the offset).
Note: It may frequently occur that the uncertainty of the offset value is due only to digitizing or the resolution of the device. If the repeatability of the offset (offset variations) is not taken into account for $u_{Anz,N}$ (because, e.g., only one measurement value is taken), it is to be taken into consideration here. Even if $p_{Offs,N} = 0$, e.g. because the offset is already deducted in the measuring instrument itself, $u_{Offs,N} > 0$.

$u_{D,N}$ uncertainty component of the offset at the time of calibration of the calibration item due to offset drift or other systematic dependencies of the offset (e.g. due to the speed dependence of spinning rotor gauges).

$u_{Cal,N}$ uncertainty component of the standard according to the calibration certificate

$u_{L,N}$ uncertainty component allowing for the long-time instability (empirical value or Type B uncertainty[2], in part statistically verified)

$u_{T,N}$ uncertainty component due to the temperature influence under the conditions of the calibration laboratory

$u_{S,N}$ uncertainty component due to the specific conditions at the calibration laboratory (e.g. different mounting position of built-in devices etc.).

The standard uncertainty of the measurement values of the standards when used at the calibration laboratory often considerably differs - due to large long-time instabilities - from the standard uncertainty at the time of their own calibration. National metrology laboratories and calibration laboratories with long-year practical experience often have empirical values which can be made use of, if appropriate.

4.2 Uncertainty contribution u_{KG} of calibration item

The uncertainty contribution of the (measurement) values of the calibration item at the time of its calibration is determined, among other things, by the resolution (analog or digital), temperature dependencies of the measuring elements (e.g. sensor, amplifier), zero point or offset variations and drift.

For the deviation of the calibration item at the time of calibration, the following model is typically valid:

$$p_{KG} = P_{Anz,KG} - p_{Offs,KG} + \delta p_{D,KG} + \delta p_{n,KG}$$

with

p_{KG} (measurement) value of calibration item
$p_{\mathrm{Anz,KG}}$ indication
$p_{Offs.KG}$ offset (zero point deviation)
$\delta p_{D,KG}$ offset drift or other systematic deviations
$\delta p_{n,KG}$ sum of deviations which are not separately stated (e.g.correction for height)

[2] see DKD-3

The standard uncertainty of the calibration item can be determined in one of the four following ways:

(a) The standard uncertainty u_{KG} of the calibration item is known.

The magnitude of u_{KG} can have been stated by the manufacturer (or applicant) as the aggregate of the uncertainty contributions and is to be used if the conditions of application specified by the manufacturer are complied with by the calibration laboratory.

(b) The dependencies or values referred to above are known.

In this case, the following relation is valid for the calculation of the standard uncertainty:

$$u_{KG} = \sqrt{u_{Anz,KG}^{2} + u_{Offs,KG}^{2} + u_{D,KG}^{2} + \sum u_{n,j,KG}^{2}}$$

with

- $u_{Anz,KG}$ uncertainty component due to insufficient repeatability (dispersion of measurement values about a mean, including uncertainty due to digitizing, resolution, etc.) *Note*: Often only one measurement value is available. The uncertainty component must then be estimated from empirical values.
- $u_{Offs,KG}$ uncertainty component of the offset at the time when the offset is measured (without repeatability of the offset). The measurement uncertainty of the offset is frequently caused solely by the resolution of the calibration item. Even if $p_{Offs} = 0$, e.g. because the offset is already deducted in the measuring instrument itself, $u_{Offs,KG} > 0$.
- $u_{D,KG}$ uncertainty component of the offset due to offset drift or other systematic dependencies (e.g. due to speed dependence in the case of spinning rotor gauges)
- $u_{n,j,KG}$ other uncertainty components not specified here, which can also stem from the calibration item, e.g. temperature influences.

(c) The dependencies or values referred to above are not known but are estimated by the calibration laboratory or empirical values for the instrument type are available.

The procedure is analogous to section (b).

(d) The above-mentioned dependencies or values

- are not known,
- cannot be estimated by the calibration laboratory, and
- manufacturer's specifications are not available.

In this case, at least two repeat measurements are to be carried out on different days. The uncertainty component of the values of the calibration item is to be determined as follows:

$$u_{KG} = u_{Rep,KG}$$

with

- $u_{Rep,KG}$ repeatability (standard deviation) of the measurement values determined for one pressure

4.3 Uncertainty contribution of calibration method

The sum δp_V of the deviations occurring during the calibration can be the result of the temperature conditions prevailing at the calibration laboratory, the uniform distribution and constancy of the adjusted pressure and the measuring method (e.g. waiting times).

Typically the following model is valid for the uncertainty contribution of the calibration method:

$$\delta p_V = \delta p_{T,V} + \delta p_{K,V} + \delta p_{M,V}$$

with

- $\delta p_{T,V}$ deviations of the pressures at the connecting flanges due to different temperatures
- $\delta p_{K,V}$ deviations of the pressures at the connecting flanges due to desorption, leaks, conditions of flow, pumping speed (e.g. in the case of cold-cathode iogauges)
- $\delta p_{M,V}$ deviations due to the measuring method (e.g. variations of the calibration pressure with time when standard and calibration item are not read simultaneously).

For the determination of the uncertainty component u_V the following is valid:

$$u_V = \sqrt{{u_{T,V}}^2 + {u_{K,V}}^2 + {u_{M,V}}^2}$$

with

- $u_{T,V}$ uncertainty component of the deviations of the pressures at the connecting flanges due to different temperatures
- $u_{K,V}$ uncertainty component of the deviations of the pressures at the connecting flanges due to desorption, leaks, conditions of flow, pumping speed
- $u_{M,V}$ uncertainty component of the deviations due to the measuring procedure.

5 Special features of the calibration of vacuum gauges

In vacuum measuring technology, the following specialties are observed:

(a) The long-time instability of the standards is often greater than their expanded uncertainty when calibrated.

(b) The measurement range of the instruments to be calibrated extends over several powers of ten. Within the scope of calibration, the indications of standard and calibration item are generally compared in a single series at increasing pressures and the deviation of the indication of the calibration item from the calibration pressure is calculated. Any repeat measurement would imply unjustifiable effort and no significant reduction of the measurement uncertainty of the calibration item when employed at the user's. Hysteresis effects are in general negligible.

(c) In some cases (e.g. Pirani gauges), it is reasonable to record the electrical output signal or an equivalent (e.g. tube constant of ionization gauge tubes) as a function of the calibration pressure.

5.1 Definition of the measurement range for Pirani gauges and statement of measurement uncertainty

The output signal of Pirani gauges often is a voltage. For the calibration this voltage is to be determined as a function of pressure. If the measured voltage is plotted in a diagram above the pressure (on the logarithmic scale), an S-shaped curve is generally obtained. The greatest steepness will more or less lie in the middle of the entire region measured. Both at the lower and at the upper end of the range, the shape will be flat. The measurement uncertainty increases extremely at the ends of the range. It is permitted to fix that range as the "measurement range" in which the slope is at least 30 percent of the maximum slope and to state a value for the measurement uncertainty for this range.

For this kind of calibration the measurement uncertainty is to be estimated unless the manufacturer has stated the uncertainty contribution of the calibration item.

6 Expansion factor *k*

It can generally be assumed that the expansion factor $k = 2$ is applicable to the calibrations.

7 Representation of the measurement uncertainty in the calibration certificate

The measurement uncertainties are most appropriately represented

- in tabular form,
- in the form of an equation,
- as a diagram,
- as a certificate of conformity.

For further details, see Guideline DKD-5.

8 Examples

8.1 Diaphragm gauge

Standard:	diaphragm gauge with digital indication, 5,5-digit
Measurement range:	0,001 mbar to 100 mbar
Calibration item (KG):	diaphragm gauge with digital indication, 4,5-digit
Measurement range:	0,01 mbar to 100 mbar
Calibration pressure (nom.):	5 mbar
Indication of standard:	5,078 mbar

Effects of calibration procedure:

Height difference of connections of standard and calibration item:	15 cm ± 1 cm
Ambient and apparatus temperature:	23,0°C ± 1°C

Uncertainty contribution u_N of standard

$u_{Anz,N}$ As an option the instrument allows the mean of the pressure measurements to be taken. As a result, the pressure indication is stabilized and the dispersion of the measurement values reduced. The uncertainty is estimated at $\pm 1 \cdot 10^{-4}$ mbar.
$u(2a) = 2 \cdot 10^{-4}$ mbar

$u_{Offs,N}$ The uncertainty of the offset values (zero point) can be calculated by the temperature coefficient for the zero point.
Temperature coefficient/zero point = 0,0004 % of FS/°C
= 0,000004·100 mbar/°C
= $4 \cdot 10^{-4}$ mbar/°C
= sensitivity coefficient c_i
(The temperature variation is ±1°C.)

$u_{D,N}$ measurement uncertainty of standard due to a drift of the zero point since its adjustment. This uncertainty is derived from the zero point fluctuations observed.
$u(2a) = 6 \cdot 10^{-4}$ mbar

$u_{Cal,N}$ uncertainty component of standard according to calibration certificate:
$u(2\sigma)$ = 0,15 % of measurement value = 0,0015·5 mbar = $7{,}3 \cdot 10^{-3}$ mbar

$u_{L,N}$ uncertainty component allowing for the long-time instability.
Empirical values of PTB: 0,1 % of meas.value u = 0,001·5 mbar = 0,005 mbar

$u_{T,N}$ uncertainty component due to the temperature influence under the conditions of the calibration laboratory
Temperature coefficient/amplification = 0,001 % of meas.value/°C
= 0,00001·5 mbar/°C
= 0,00005 mbar/°C
= sensitivity coefficient c_i
(The temperature variation amounts to ±1°C.)

$u_{S,N}$ uncertainty component due to the special conditions at the calibration laboratory. Other effects on the measurement uncertainty are not known. This uncertainty component therefore is = 0.

Uncertainty contribution u_{KG} of calibration item

$u_{Anz,KG}$ uncertainty component due to insufficient repeatability.
Manufacturer's specification: 0,08 % of meas.value
$u(2\sigma)$ = 0,0008·5 mbar = 0,004 mbar

$u_{Offs,KG}$ uncertainty component of offset at the time the offset is measured. This uncertainty is essentially due to the digital resolution:
$u(2a) = 2 \cdot 10^{-3}$ mbar

$u_{D,KG}$ uncertainty of offset at the time of calibration due to offset drift. The uncertainty of the offset values (zero point) can be calculated through the temperature coefficient for the zero point.
Temperature coefficient/zero point = 0,002 % of FS/°C
= 0,00002·100 mbar/°C
= $2 \cdot 10^{-3}$ mbar/°C
= sensitivity coefficient c_i
(The temperature variation amounts to ±1°C.)

$u_{T,KG}$ uncertainty contribution due to the temperature influence.
Temperature coefficient/span = 0,01 % of meas.value/°C
= 0,0001·5 mbar/°C
= 0,0005 mbar/°C
= sensitivity coefficient c_i

Uncertainty contribution of calibration procedure

$u_{T,V}$ uncertainty component of the deviation of the pressures at the connecting flanges due to different temperatures. The difference of the gas temperatures at the connecting flanges is estimated at max. 0,02°C (due to air flows). As the volume is constant, the resulting pressure difference can be calculated by Gay-Lussac's law.

$p/p_0 = T/T_0$ = 296,17°C/296,15°C = 1,000068,
i.e. 0,0068 % of p/°C = 0,00034 mbar/°C = sensitivity coefficient c_i

(Hint: This uncertainty component does not have a correction component but makes a contribution to the measurement uncertainty.)

$u_{K,V}$ uncertainty component due to the hydrostatic pressure difference in case the connecting flanges are at different heights:

	unit
$\Delta p = p \cdot g \cdot h$	
$= 1{,}27 \cdot 10^{-3} \cdot (p/\text{mbar}) \cdot 9{,}81 \cdot (h/\text{m})$	kg m^{-3} m s^{-2} m
$= 1{,}2 \cdot 10^{2} \cdot (p/\text{mbar}) \cdot 9{,}81 \cdot (h/\text{m})$	kg m s^{-2} m^{-2}
$= 1{,}2 \cdot 10^{-2} \cdot (p/\text{mbar}) \cdot (h/\text{m})$	kg m s^{-2} m^{-2}
$= 1{,}2 \cdot 10^{-2} \cdot (p/\text{mbar}) \cdot (h/\text{m})$	N m^{-2}
$= 1{,}2 \cdot 10^{-4} \cdot (p/\text{mbar}) \cdot (h/\text{m})$	Pa · 0,01 mbar / Pa
$= 1{,}2 \cdot 10^{-4} \cdot (p/\text{mbar}) \cdot (h/\text{m})$	mbar

with

$\rho = \rho_0 \cdot p/p_0$ = 1,2929 kg·m^{-3} ·p/1013 mbar = 1,27·10^{-3} ·(p/mbar) kg m^{-3}

with

ρ_0 = 1,2929 kg m^{-3} (273,15 °K, 1013,25 mbar)

(Hint: For the calculation of $\Delta\rho$ the pressure p must be taken in mbar and the height h in m. The resulting numerical value for $\Delta\rho$ then is given in mbar.)

The deviation at 5 mbar and a height difference of 0,15 m is:
$\Delta\rho$ = 1,2·10^{-4} ·(5 mbar/mbar)·(0,15 m/m) = 0,00009 mbar.

The sensitivity coefficient
$c_i = \rho \cdot g$ = 1,2·10^{-4} ·(ρ/mbar) mbar m^{-1}.

At a measuring pressure of 5 mbar, the sensitivity coefficient c_i = 0,0006 mbar/m.
The uncertainty of the height difference is ±0,01 m.

$u_{M,V}$ uncertainty component of the deviations due to the measuring method.
At a leakage rate of 5·10^{-6} mbar·l/s, the pressure in the tank (20 l) rises in 20 s by 5·10^{-6} mbar. This change in pressure is not used as correction but makes a contribution to the measurement uncertainty.

Uncertainty budget for a diaphragm gauge at a calibration pressure of 5 mbar:

Quantity	Estimate value	Width of distribution		Distribution*)	Divider	Uncertainty		Sensitivity coefficient		Uncertainty contribution	Index
X_i	x_i	$2a$				$u(x_i)$		c_i		$u_i(y)$	
	mbar									mbar	%
$p_{Anz,N}$	5,078	2,00E-04	mbar	R	$2\cdot\sqrt{3}$	5,77E-05	mbar	1,0		5,77E-05	0,0
$p_{Offs,N}$	0	2	°C	R	$2\cdot\sqrt{3}$	5,77E-01	°C	4,0E-04	mbar/°C	2,31E-04	0,2
$\delta p_{D,N}$	0	6,00E-04	mbar	R	$2\cdot\sqrt{3}$	1,73E-04	mbar	1,0		1,73E-04	0,1
$\delta p_{Cal,N}$	-0,003	7,30E-03	mbar	N	2	3,65E-03	mbar	1,0		3,65E-03	48,5
$\delta p_{L,N}$	0	1,00E-02	mbar	R	$2\cdot\sqrt{3}$	2,89E-03	mbar	1,0		2,89E-03	30,3
$\delta p_{T,N}$	0	2	°C	R	$2\cdot\sqrt{3}$	5,77E-01	°C	5,0E-05	mbar/°C	2,89E-05	0,0
$\delta p_{S,N}$	0	0						1,0		0	0,0
$\boldsymbol{p_N}$	**5,075**									**0,0047**	**79,1**
$p_{Anz,KG}$	5,140	0,004	mbar	N	2	2,00E-03	mbar	1,0		2,00E-03	14,5
$p_{Offs,KG}$	0	2,00E-03	mbar	R	$2\cdot\sqrt{3}$	5,77E-04	mbar	1,0		5,77E-04	1,2
$\delta p_{D,KG}$	0	2	°C	R	$2\cdot\sqrt{3}$	5,77E-01	°C	2,0E-03	mbar/°C	1,15E-03	4,8
$\delta p_{T,KG}$	0	2	°C	R	$2\cdot\sqrt{3}$	5,77E-01	°C	5,0E-04	mbar/°C	2,89E-04	0,3
$\boldsymbol{p_{KG}}$	**5,140**									**0,0024**	**20,9**
$\delta_{T,V}$	0	0,02	°C	R	$2\cdot\sqrt{3}$	5,77E-03	°C	3,4E-04	mbar/°C	1,96E-06	**0,0**
$\delta p_{K,V}$	-9.0E-05	0,02	m	R	$2\cdot\sqrt{3}$	5,77E-03	m	6,0E-04	mbar/m	3,46E-06	**0,0**
$\delta p_{M,V}$	0	5,00E-06	mbar	R	$2\cdot\sqrt{3}$	1,44E-06	mbar	1,0		1,44E-06	**0,0**
$\boldsymbol{\delta p_V}$	**-9,0E-05**									**0,00000**	**0,0**
$\boldsymbol{\Delta p}$	**0,0649**	**Expanded uncertainty $U = k \cdot u$ (k = 2):**								**0,0106**	**100,0**

*) R - rectangular distribution
N - normal distribution

Result:

The deviation of the calibration item thus is:

Δp = 0,0649 mbar ± 0,0106 mbar

Note: The values given in the "Index" column give the share of the partial uncertainties in the overall uncertainty in percent and illustrate the weighting of the individual influence quantities. Their calculation is not mandatory but it shows where to begin when searching ways to reduce the measurement uncertainty.

8.2 Pirani gauge

Standard:	diaphragm gauge with digital indication, 5,5-digit
Measurement range:	0,00001 mbar to 1 mbar
Calibration item (KG):	Pirani gauge with digital indication, 4,5-digit
Measurement range:	0,001 mbar to 100 mbar
Calibration pressure:	0,2 mbar

Effects of calibration procedure:

Height difference of connections of standard and calibration item:	0 cm ± 1 cm
Ambient and apparatus temperature:	23,0°C ± 1°C

Uncertainty contribution u_N of standard

$u_{Anz,N}$ As an option the instrument allows the mean of the pressure measurements to be taken. As a result, the pressure indication is stabilized and the scatter of the measurement values reduced. The uncertainty is estimated at $\pm 1 \cdot 10^{-5}$.
$u(2a) = 2 \cdot 10^{-5}$ mbar

$u_{Offs,N}$ The uncertainty of the offset values (zero point) can be calculated by the temperature coefficient for the zero point.
Temperature coefficient/zero point = 0,0004 % of FS/°C
= 0,000004·1 mbar/°C
= $4 \cdot 10^{-6}$ mbar/°C
= sensitivity coefficient c_i
(The temperature variation is ±1°C.)

$u_{D,N}$ measurement uncertainty of standard due to a drift of the zero point since its adjustment. This uncertainty is derived from the zero point fluctuations observed.
$u(2a) = 6 \cdot 10^{-6}$ mbar

$u_{Cal,N}$ uncertainty component of standard according to calibration certificate:
$u(2\sigma)$ = 0,3% of measurement value = 0,003 · 0,2 mbar = $6 \cdot 10^{-4}$ mbar

$u_{L,N}$ uncertainty component allowing for the long-time instability.
Empirical values of PTB: 0,35 % of meas.value
$u(2a)$ = 0,0035 · 0,2 mbar = $7 \cdot 10^{-4}$ mbar

$u_{T,N}$ uncertainty component due to the temperature influence under the conditions of the calibration laboratory.
Temperature coefficient/amplification = 0,001 % of meas.value/°C
= 0,00001 · 0,2 mbar/°C
= 0,000002 mbar/°C
= sensitivity coefficient c_i
(The temperature variation amounts to ±1°C.)

$u_{S,N}$ uncertainty component due to the special conditions at the calibration laboratory. Other influences on the measurement uncertainty are not known. This uncertainty component therefore is = 0.

Uncertainty contribution u_{KG} of calibration item

$u_{Anz,KG}$ uncertainty component due to insufficient repeatability. The empirical value for this gauge is 2 % of meas.value
$u(2a) = 0{,}02 \cdot 0{,}2$ mbar = 0,004 mbar

$u_{Offs,KG}$ uncertainty component of offset at the time the offset is measured. This uncertainty is essentially due to the digital resolution:
$u(2a) = 2{\cdot}10^{-3}$ mbar

$u_{D,KG}$ uncertainty of offset is estimated at $u(2a) = 2{\cdot}10^{-4}$.

$u_{T,KG}$ uncertainty contribution due to the temperature influence.
Temperature coefficient/span = 3 % of meas.value/°C
= 0,03 · 0,2 mbar/°C
= 0,006 mbar/°C
= sensitivity coefficient c_i
(The temperature variation amounts to ±1°C.)

Uncertainty contribution of calibration method

$u_{T,V}$ uncertainty component of the deviation of the pressures at the connecting flanges due to different temperatures. The difference of the gas temperatures at the connecting flanges is estimated at max. 0,02°C (due to air flows). As the volume is constant, the resulting pressure difference can be calculated by Gay-Lussac's law.

$p/p_0 = T/T_0$ = 296,17°C/296,15°C = 1,000068,
i.e. 0,0068 % of p/°C = 0,000014 mbar/°C
= sensitivity coefficient c_i

(Hint: This uncertainty component does not have a correction component but makes a contribution to the measurement uncertainty.)

$u_{K,V}$ uncertainty component due to the hydrostatic pressure difference in case the connecting flanges are at different heights:

$\Delta p = p \cdot g \cdot h$	unit
= 1,27·10-3 ·(p/mbar) · 9,81 ·(h/m)	kg m-3 m s-2 m
= 1,2·10-2 ·(p/mbar) ·(h/m)	kg m s-2 m-2
= 1,2·10-2 ·(p/mbar) ·(h / m)	N m-2
= 1,2 10-4 ·(p/mbar) ·(h/m)	Pa ·0,01 mbar/Pa
= 1,2 10-4 ·(p/mbar) ·(h/m)	mbar

with $\rho = \rho_0 \cdot p/p0$ = 1,2929 kg m^{-3} ·p/1013 mbar = 1,27·10-3 ·(p/mbar) kg m^{-3}

with ρ_0 = 1,2929 kg m^{-3} (273,15 °K; 1013,25 mbar)

(Hint: For the calculation of Δp the pressure p must be taken in mbar and the height h in m. The resulting numerical value for Δp then is given in mbar.)
The sensitivity coefficient $c_i = p \cdot g$
$= 1{,}2{\cdot}10^{-4} \cdot (p/\text{mbar})$ mbar m^{-1}.

At a measuring pressure of 0,2 mbar, the sensitivity coefficient c_i = 0,000024 mbar/m.
The uncertainty of the height difference is ±0,01 m.

$u_{M,V}$ uncertainty component of the deviations due to the measuring method.
At a leakage rate of $5{\cdot}10^{-6}$ mbar·l/s, the pressure in the tank (20 l) rises in 20 s by $5{\cdot}10^{-6}$ mbar. This change in pressure is not used as correction but makes a contribution to the measurement uncertainty.

Uncertainty budget for a Pirani gauge at a calibration pressure of 0,2 mbar:

Quantity	Estimate value	Width of distribution		Distribution*)	Divider	Uncertainty		Sensitivity coefficient		Uncertainty contribution	Index
X_i	x_i	$2a$				$u(x_i)$		c_i		$u_i(y)$	
	mbar									mbar	%
$p_{Anz,N}$	0,20077	2,00E-05	mbar	R	$2\cdot\sqrt{3}$	5,77E-06	mbar	1,0		5,77E-06	0,0
$p_{Offs,N}$	0	2	°C	R	$2\cdot\sqrt{3}$	5,77E-01	°C	4,0E-06	mbar/°C	2,31E-06	0,0
$\delta p_{D,N}$	0	6,00E-06	mbar	R	$2\cdot\sqrt{3}$	1,73E-06	mbar	1,0		1,73E-06	0,0
$\delta p_{Cal,N}$	-0,00156	6,00E-04	mbar	N	2	3,00E-04	mbar	1,0		3,00E-04	0,5
$\delta p_{L,N}$	0	0,0007	mbar	R	$2\cdot\sqrt{3}$	2,02E-04	mbar	1,0		2,02E-04	0,2
$\delta p_{T,N}$	0	2	°C	R	$2\cdot\sqrt{3}$	5,77E-01	°C	2,0E-06	mbar/°C	1,15E-06	0,0
$\delta p_{S,N}$	0	0								0	0,0
$\boldsymbol{p_N}$	**0,19921**									**0,00036**	**0,8**
$p_{Anz,KG}$	0,200	0,004	mbar	N	2	2,00E-03	mbar	1,0		2,00E-03	24,3
$p_{Offs,KG}$	0	2,00E-03	mbar	R	$2\cdot\sqrt{3}$	5,77E-04	mbar	1,0		5,77E-04	2,0
$\delta p_{D,KG}$	0	0,0002	mbar	R	$2\cdot\sqrt{3}$	5,77E-05	mbar	1,0		5,77E-05	0,0
$\delta p_{T,KG}$	0	2	°C	R	$2\cdot\sqrt{3}$	5,77E-01	°C	6,0E-03	mbar/°C	3,46E-03	72,9
$\boldsymbol{p_{KG}}$	**0,200**									**0,0040**	**99,2**
$\delta_{T,V}$	0	0,02	°C	R	$2\cdot\sqrt{3}$	5,77E-03	°C	1,4E-05	mbar/°C	8,083E-08	**0,0**
$\delta p_{K,V}$	0	0,02	m	R	$2\cdot\sqrt{3}$	5,77E-03	m	2,4E-05	mbar/m	1,386E-07	**0,0**
$\delta p_{M,V}$	0	5,00E-06	mbar	R	$2\cdot\sqrt{3}$	1,44E-06	mbar	1,0		1,443E-06	**0,0**
$\boldsymbol{\delta p_V}$	**0**									**0,00000**	**0,0**
$\boldsymbol{\Delta p}$	**0,00079**	**Expanded uncertainty $U = k \cdot u$ (k = 2):**								**0,0080**	**100,0**

*) R - rectangular distribution
N - normal distribution

Result:

The deviation of the calibration item thus is:

Δp = 0,00079 mbar ± 0,0080 mbar

Note: The values given in the "Index" column give the share of the partial uncertainties in the overall uncertainty in percent and illustrate the weighting of the individual influence quantities. Their calculation is not mandatory but it shows where to begin when searching ways to reduce the measurement uncertainty.

References

The measurement uncertainty analysis is based on the following documents:

ISO:
Guide to the Expression of Uncertainty in Measurement
1st ed. 1993, ISO, Geneva, Switzerland, ISBN 92-67-10188-9

DIN V ENV 13005:
Leitfaden zur Angabe der Unsicherheit beim Messen
German version ENV 13005:1999

EA-4/02:
Expression of the Uncertainty of Measurement in Calibration
December 1999

DKD-3:
Angabe der Messunsicherheit bei Kalibrierungen
DKD 1998, German version of the publication EA-4/02: *Expression of the Uncertainty of Measurement in Calibration*

DKD-3-E1*:* Supplement 1
Angabe der Messunsicherheiten bei Kalibrierungen, Beispiele
DKD 1998, German version of the publication EA-4/02-S1: *Expression of the Uncertainty of Measurement in Calibration,* Examples

DIN 1319-3:
Grundlagen der Messtechnik, Teil 3: Auswertung von Messungen einer einzelnen Messgröße, Messunsicherheit, 1996

For further standards and references, see Part 1: Fundamentals.